BOOK REVIEWS

Mathematical Statistics for Economics and Business, RON C. MITTELHAMMER, Springer-Verlag, New York, 1996, xviii + 723 pp. $57·95, hardback only; solutions manual free to instructors who adopt the text.

Recent years have seen an increasingly sophisticated set of estimation, testing and evaluation methods enter the applied economist's toolkit. The level of statistical competency required of the applied economist has increased concomitantly. If these statistical topics are covered in an econometrics course, either econometrics or statistics receives insufficient attention. Taking the standard first-year graduate course in the statistics department is not an efficient option: too much time is spent on topics not relevant to econometrics and not enough time is spent on those topics which are relevant. Therefore, some PhD programmes offer a semester of 'statistics for econometrics' (SFE) prior to the first econometrics course. Perusing their syllabi makes clear the need for a book such as the one reviewed here.

These SFE courses all rely on a standard, introductory graduate statistics textbook such as Hogg and Craig (1994). Such textbooks often presume a degree of mathematical sophistication not possessed by many first-year graduate students in economics. Only a portion of the textbook is relevant to econometrics, so many of the topics in the textbook are not covered. That portion which is relevant is not treated in nearly enough detail. There are many statistical topics relevant to econometrics which are not covered in the text. Therefore, many supplementary readings may be required. Naturally, the text, its exercises, and possibly the supplementary readings are not written with econometrics in mind. Consequently, as far as economics is concerned, the material is neither unified nor integrated and may be judged marginally adequate, at best. Even upon successful completion of such a course, the student may well wonder what relevance his newly acquired knowledge has for econometrics. All this can make it difficult not only to take such a course but to teach such a course as well. Given these twin difficulties, it is not surprising that SFE is not offered by more schools, but Mittelhammer's contribution can ameliorate the situation. This fine text largely remedies many of the above defects, and will as much improve the taking of the course as the teaching of it.

The preface begins 'This book is designed to provide beginning graduate students with a rigorous and accessible foundation in the principles of probability and mathematical statistics underlying statistical inference in the fields of business and economics'. My impression is that 'in the fields of business and economics' is a bit too broad. I doubt that many MBA programmes require such a thorough knowledge of statistics. Moreover, ANOVA is not covered, which might trouble professors from a business curriculum but not economics professors. However, I am certain that 'in the field of econometrics' is not too narrow, and that this book is well suited to the needs of economists who aspire to do competent econometric work. Disconcertingly, there are some typos and misprints, and not all of them are innocuous. Some of them affect formulae and chapter exercises. Fortunately, Mittelhammer has an errata list on his homepage (www.agecon.wsu.edu/faculty/mittelhammer/index.htm). Also, the book does have a grammatical flaw. While the gender-neutral use of the masculine pronoun has been almost completely eliminated in favour of the feminine pronominal form (Campbell and Campbell-Wright, 1995), I did find one use of the word 'he' (p. 168). These minor quibbles aside, Mittelhammer admirably attains his stated goals.

The book is rigorous and accessible. Only rarely are proofs omitted, (e.g., completeness in the exponential class, a few central limit theorems), and it seems that such proofs are omitted more because

their marginal pedagogical values are nil than because they are too complicated. Generally, the proofs are much more detailed than found in a typical statistics text. This can be useful for students not already familiar with the theorem/proof format of rigorous mathematics courses. Just as important are the extensive discussions and examples which commonly follow the proofs. These discussions focus on the intuition underlying the theorem and its proof, and are reminiscent of some of the better pages of Cox and Hinkley (1974). The examples often are of an economic or econometric nature. Consider Mittelhammer's proof of the Lindberg–Levy central limit theorem (L–L CLT, p. 270) which takes nearly a full page. Immediately following is a two-page discussion of the intuition behind the L–L CLT. This, in turn, is followed by two examples of applying the L–L CLT, which take an additional two pages. By contrast, Amemiya (1985) proves the L–L CLT in six lines with neither discussion nor examples. Ramanathan (1993) proves it in half a page, but offers neither discussion nor examples. Hogg and Craig omit this theorem completely. Mittelhammer's treatment of the L–L CLT is not an exception, but a rule consistently applied throughout the book: give a detailed proof of a theorem, discuss its intuition, and provide examples of applying the theorem. Definitions, too, frequently are followed by intuition and examples. For example, the definition of the asymptotic distribution of a convergent sequence (p. 239) is followed by a half-page discussion and then by a page of examples.

Two further points illustrate the rigour of the book. First, many econometrics texts only give one definition of asymptotic unbiasedness (a.u.) and do not adequately treat the implications of consistency for a.u. Mittelhammer (pp. 385–6) gives two definitions of a.u. and even provides an example which illustrates the difference between a.u. and consistency. Second, many textbooks treat the 'size' and 'level' of a test as though they were the same thing. Mittelhammer carefully distinguishes between the two (pp. 531–533).

One obvious omission, doubtless intentional on the part of the author, is that he does not take a measure-theoretic approach to probability. Consequently, filtrations as information sets and their effect on conditioning are not explored. Since the book emphasizes the statistical knowledge which is necessary to understand applied econometric methods, the omission does no great damage, though specialists in financial econometrics would disagree. However, professors who believe that the measure-theoretic approach is an absolute necessity can easily add Gallant's (1997) superb (and slim) monograph to the reading list.

Much of the contents of each chapter is fairly standard, so there is no need to recount in detail the topics covered by each chapter. However, each chapter contains a few pleasant surprises of which note should be taken. That I do not mention a specific topic is not to suggest that the topic is not covered.

Chapter One (42 pp.), 'Elements of Probability', makes some use of measure-theoretic devices, including Borel sets and probability spaces. Some theorems and exercises make use of the probability space.

Chapter Two (66 pp.), 'Random Variables, Densities, and Cumulative Distribution Functions', makes more use of the probability space and defines an induced probability space. This use of measure-theoretic notions is largely heuristic, and its purpose seems to be twofold: first, to acquaint the reader with these important topics; and second, to set up the proper definition of conditional expectation in a later chapter. Mixed discrete-continuous distributions are also covered, which is a topic missing from many texts.

Chapter Three (60 pp.), 'Mathematical Expectation and Moments', stresses the role of conditional expectation in regression, and explicitly defines the regression function as a conditional expectation. In less-than-rigorous texts it is not uncommon to see conditional expectation explicitly defined for the discrete case, followed by much hand-waving in the continuous case. However, since the previous chapters introduced sufficient measure-theoretic ideas, this chapter can give an accurate definition of conditional expectation in the continuous case. Most unfortunately, skewness is barely mentioned and kurtosis is mentioned not at all. Until the second edition comes out, these things are easy enough for the professor to take care of. This chapter also contains a lucid treatment of the moment-generating function (MGF) and its uses, at a much more practical level than usually is found in statistical texts, e.g., Hogg and Craig. I was quite surprised to find that the use of the characteristic function (CF) was excluded in favour of the MGF. The typical reason a textbook gives for using the MGF instead of the CF is because it is easier on the student. While Mittelhammer does express a desire to avoid complex numbers, a course on statistics cannot be fully rigorous without including the CF.

Chapter Four (52 pp.), 'Parametric Families of Density Functions', includes not only the usual suspects but the gamma and beta distributions, as well as the exponential class (non-central distributions are deferred until Chapters 9 and 10). Partitioned inversion and partitioned determinants of matrices are introduced (with examples, naturally), so the student already is exposed to them when they recur in the context of the linear model.

Chapter Five (78 pp.), 'Basic Asymptotics', has an even better discussion of the various modes of convergence than Spanos (1986) and includes both triangular array convergence and a CLT for bounded M-dependent processes. As usual, examples and discussions of intuition abound. Any professor who teaches *Asymptotic Theory for Econometricians* (White, 1986) should assign this chapter as a prerequisite. In a refreshing change from the usual exercises associated with this topic, Mittelhammer's exercises are almost all applied in nature, giving the student concrete examples of what the asymptotic theorems can do. Many texts ignore that recurring asymptotic question, 'How large does 'n' have to be?' Mittelhammer rightly considers the Berry–Esseen Inequality as sharpened by Beeck, gives an example of its use and even asks the student to use it in three of the chapter exercises. When I read the first of these exercises, I knew that Mittelhammer had presented a bound and I simply assumed it was the Berry–Esseen Inequality, but I did not recall seeing the words 'Berry–Esseen'. These words are not in the index. I finally found them in a footnote (p. 273) which only cites Beeck's article — the words occur in the title of the article. Doubtless many students will ignore footnotes which only cite articles and fruitlessly scour the text searching for the words 'Berry–Esseen'. Fortunately, Mittelhammer makes few such oversights.

Chapter Six (66 pp.), 'Sampling, Sample Moments, Sampling Distributions, and Simulation', includes five pages on empirical distributions which conclude with the Glivenko–Cantelli Theorem. The penultimate section of this chapter is entitled 'Random Sample Simulation and the Probability Integral Transform'. These, and other pages scattered throughout the text, (e.g., a discussion of pivotal quantities) ensure that this text would be a suitable prelude to an econometrics text which includes a chapter on the bootstrap.

Chapter Seven (64 pp.), 'Elements of Point Estimation Theory', opens with a discussion of 'statistical models' and 'parameter identifiability', but does not take the next step to 'data generating process'. Perhaps expecting this betrays my fondness for the London School of Economics (LSE) methodology as typified by Spanos (1986) and Hendry (1995). Since the concept of a data generating process is finally creeping into mainstream econometrics texts (Johnston and DiNardo, 1997, p. 250; Davidson and MacKinnon, 1993, p. 53; Greene, 1997, p. 846), it certainly merit mentions in a text which lays the statistical foundations for the study of econometrics. Sufficiently, minimal sufficiency, and Rao–Blackwell are treated, though ancillarity is nowhere to be found. This lapse of rigour is regrettable, since ancillarity is the dual of sufficiency. There are several pages of MVUE results, much of which is dedicated to the Cramer–Rao lower bound in its various guises.

Chapter Eight (82 pp.), 'Point Estimation Methods', focuses on the 'general linear model' (GLM) and carefully distinguishes between two cases when X can be considered fixed: designed experiments, and y conditional on x. The discussion of the latter case emphasizes that $y = X\beta + e$ really means $y|x = x\beta + e|x$, and that a critical assumption underlying the GLM is not $E[e] = 0$ but $E[e \,|\, x] = 0$. These delicate but important points are overlooked in many texts, and failure to appreciate them has ruined many a simulation study. The proof of the consistency of $\hat{\beta}$ takes a quarter-page, followed by a two-page discussion of the proof which novice and expert alike may find greatly illuminating.

Chapter Nine (86 pp.), 'Elements of Hypothesis Testing Theory', explicitly defines the useful concepts of an admissible test, a consistent sequence of tests, and asymptotic tests. It also includes a long section, 'Results on UMP Tests'. The discussion of the Neyman–Pearson Lemma is two pages, followed by two detailed examples covering three and a half pages. Nuisance parameters are discussed in the subsection 'Conditioning in the Multiple Parameter Case' but, again, ancillarity is noticeably absent.

Chapter Ten (82 pp.), 'Hypothesis Testing Methods', includes the generalized likelihood ratio, Lagrange Multiplier, and Wald Tests. Some texts treat all three methods in a single section, perhaps stating a single theorem and giving one example for each test. Mittelhammer devotes an entire section to each method, as usual stating the theorem, giving a detailed proof with discussion, and including several examples.

Moreover, for each method he also states (and proves) the corresponding theorem under local alternatives when the null is false (the case of 'Pitman drift'). Confidence intervals are considered from two directions: as the dual of an hypothesis test and also as an extension of pivotal quantities. The chapter concludes with a survey of some non-parametric methods, including not only the Kolmogorov–Smirnoff test, but also the Lilliefors extension of that test to the null of normality.

On the recommendation of a friend, I bought this book when it first came out. I read it, found it quite enlightening on a number of topics, and considered it a good text. Recently I had occasion to refer to it and, to enhance my understanding of the point in question, worked the relevant chapter exercises. Upon completion of this task, I decided the book was not good, but great, and I decided to work through all the exercises.

There is an average of about 20 exercises at the end of each chapter, the vast majority of them word problems, most with multiple parts. Frequently exercises in one chapter draw on results from previous chapters, requiring the student to keep in mind that which he already has learned. At this writing, I have worked through all the exercises in the first six chapters at about three to four hours per chapter, though Chapter Five took somewhat longer. *Mathematica* (Wolfram, 1996), which I used for integration, matrix manipulation, and other sundry activities, saved me a not insubstantial amount of time. I did observe in Chapters Five and Six that some of the exercises might be exceedingly difficult—perhaps impossible—for the novice to solve, unless the professor provides some specific direction or hints. Such exercises might also exist in the remaining chapters. The professor would do well to warn students of such problems, so that the students do not become unnecessarily discouraged. In a later edition, the author might wish to mark such exercises with an asterisk. Additionally, a few exercises require non-statistical knowledge which is not presented in the text. One such exercise was number 11 of Chapter 3, which required me to refresh my knowledge of efficient frontier portfolios, a topic not discussed in the text. However, the professor can easily provide this knowledge to the class in a few minutes, before assigning the exercise.

My own formal training in statistics was a year-long graduate course, in a statistics department, covering Hogg and Craig. Anyone with similar training or who has taught SFE will have little trouble with the exercises. Moreover, the 173-page solutions manual appears to be quite good. It is written with an economy of style directed more toward the teacher than the student, but the solutions are clear and straightforward and are not of the irritating type where 'obvious' steps are omitted for brevity.

Were I to teach a 'statistics for econometrics' course, I should adopt this text without hesitation. I strongly urge any current or prospective teacher of such a course to consider this text, for this is precisely the type of book I wish had been available when I was in graduate school. Even ten years later and with the benefit of Hogg and Craig under my belt, reading this book and working through the exercises has been enormously edifying. An economist who wants to sharpen the rough edges of his statistical knowledge would do well to work through this book, but there is a caveat: if you never took the year-long graduate course in statistics, first be sure to beg, borrow, or steal a copy of the solutions manual.

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